Where Common Knowledge Cannot Be Formed, Common Belief Can – Planning with Multi-Agent Belief Using Group Justified Perspectives

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1 Introduction and Motivation

Epistemic planning is a sophisticated branch of automated planning that integrates elements from both classical planning and epistemic logic. It allows agents to reason about not only the physical world but also others' knowledge and beliefs. It is suitable for solving multi-agent cooperative or adversarial tasks.

Research by either explicitly maintaining all epistemic relations [16, 2, 1] or requiring an expensive pre-compilation step to convert an epistemic planning problem into a classical planning problem [21, 5] faces exponential growth in terms of the depth of the epistemic formulae. Recently, a lazy state-based approach called *Planning with Perspectives* (PWP) [13] is proposed that uses F-STRIPS [9] to reason about the agent's seeing relation and knowledge. Their intuition is that an agent knows a proposition if it can see the variables involved in the proposition, and that proposition is true. As an extension to handle beliefs, the *Justified Perspective* (JP) [14] model is proposed to handle beliefs (as well as knowledge), permitting e.g. conflicting beliefs between agents. However, the JP model could only reason about individual's nested belief, not group belief operators such as common belief.

Here, we extend the JP model to model uniform belief, distributed belief, and common belief. However, applying this intuition of "belief is past knowledge" naïvely to group belief is neither complete nor consistent. It is possible to form a common belief about a proposition even if there was no prior common knowledge about this. We illustrate this idea by extending the false-belief example from [14].

Example 1. There are two agents a and b, and there is a number $n \in \mathbb{N}$ inside a box. The number can only be seen by the agents when they are peeking into the box. The agents know whether the others are peeking into the box. The actions that agents can do are: *peek* and *return*. There are two hidden actions *add* and *subtract* (performed by another hidden agent), and their effects are only visible to the agents who are peeking into the box. Initially, both agents a and b are not peeking, and the value of n is 2. The task is to generate a plan such that: the common belief between a and b is $n<3$.

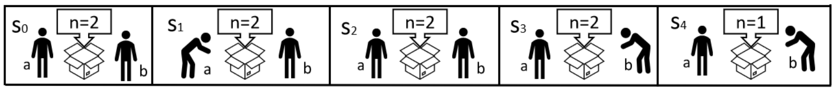

Fig. 1: An example plan.

A valid plan (Figure 1) to achieve the common group belief above would be: $peek(a)$, $return(a)$, $subtract$, $peek(b)$. Agents a and b do not peek at the same time. So, at no point does neither the statement "agent a knows that agent b knows $n{<}3$" ($K_a K_b n{<}3$) nor $K_b K_a n{<}3$ hold – an agent only knows something if they can see it. Further, the common knowledge $CK_{\{a,b\}} n{<}3$ does not hold.

However, we assert that the common belief $CB_{\{a,b\}} n{<}3$ should hold if agents have memory. Since agent a sees $n{=}2$ after step 1 and agent b sees $n{=}1$ after step 4, both $B_a n{=}2$ and $B_b n{=}1$ hold, which implies both $B_a n{<}3$ and $B_b n{<}3$. In addition, since agent a sees agent b peeking into the box after step 4 and $B_a n{=}2$, $B_a B_b n{=}2$ should hold. Similarly, $B_b B_a n{=}1$ should hold. Therefore, we have both $B_a B_b n < 3$ and $B_b B_a n < 3$. Given that a and b both saw that each other peeked in the box, and saw that each saw that each peeked into the box, etc, both a and b believe each other believes $n{<}3$ with infinite depth. From the definition by Fagin et al. [7], this constitutes common belief.

2 Background

For group beliefs, there are mainly three types: uniform beliefs (shared beliefs); distributed beliefs; and common beliefs. This paper follows the standard definitions in existing research work for uniform beliefs [15, 8] and common beliefs [20, 23, 3, 12, 4], while distributed beliefs require some more clarification.

Distributed belief, denoted $DB_G\varphi$, combines the beliefs of all agents in group G. It is, effectively, the pooled beliefs of group G if the agents were to "communicate" everything they believe to each other. This is challenging since agents may hold different beliefs. There are two main approaches to model distributed belief: (1) belief merging [17, 18, 6]; and (2) merging the agents' epistemic accessibility relations [11, 26, 22, 27, 24, 19, 10, 25]. Typically, merging conflicting beliefs is solved using some form of ordering over agents or propositions.Here, we propose a definition for group distributed justified belief, in which conflicting beliefs are removed entirely, leading to a modal operator that obeys the axiom of consistency (axiom D) while dropping Axiom D2 ($D_G\varphi \rightarrow D_{G'}\varphi$ if $G \subseteq G'$).

3 Group Justified Perspective Model

In this section, we define group justified perspective functions for uniform belief EB, distributed belief DB, and common belief CB, and add ternary semantics for them. Then, we show[3] in our semantics that EB, DB, and CB satisfy the

[3]A complete version of this paper can be found here: https://arxiv.org/abs/2412.07981

axioms **KD**, **KD45** , and **KD4** , respectively. Due to the page limit, we omit EB as its definition, semantics, and theorems are straightforward.

The definition of the signature Σ=$(Agt, V, \mathbb{D}, \mathcal{R})$, language $\mathcal{L}_{GKB}(\Sigma)$, states (sets of variable assignments, $\mathcal{S}$ and $\mathcal{S}_c$ as state spaces and complete-state space), and model instance $M = (Agt, V, \mathbb{D}, \pi, O_1, \ldots, O_k)$ are adopted from the JP model [14], as well as three functions: observation function ($O_i : \mathcal{S} \to \mathcal{S}$, which should be contractive, idempotent, and monotonic), retrieval function ($R : \overrightarrow{\mathcal{S}} \times \mathbb{Z} \times V \to \mathbb{D}$) and Justified Perspective (JP) function ($f_i : \overrightarrow{\mathcal{S}_c} \to \overrightarrow{\mathcal{S}_c}$). Their intuition is that agents reason about beliefs by constructing a justified perspective (state sequence) according to their own observation (O_i), and for the parts they are not observing, they retrieve the most recent observation (R).

Distributed Belief is more challenging compared to distributed knowledge. If we simply take the distributed union of the perspectives for all agents $i \in G$, the generated set could contain conflicting (inconsistent) beliefs. To ensure consistency, we form the group distributed justified perspective instead of just uniting each agent's justified perspective. Intuitively, agents follow their own observations and "listen" to agents that have seen variables more recently.

Definition 1 (Distributed Justified Perspectives). *The distributed justified perspective function for a group of agents G is defined as follows:*

$$df_G([s_0, \ldots, s_n]) = [s'_0, \ldots, s'_n]$$

where for all $t \in [0, n]$ *and all* $v \in dom(s_t)$*:*

$$\begin{aligned}
lt_{tv} &= \max(\{j \mid v \in \textstyle\bigcup_{i \in G} O_i(s_j) \wedge j \leq t\} \cup \{-1\}), && (1)\\
e &= R([s_0, \ldots, s_t], lt_{tv}, v), && (2)\\
s''_t &= \{v = e \mid s_t(v) = e \vee v \notin \textstyle\bigcup_{i \in G} O_i(s_t\langle\{v = e\}\rangle)\}, && (3)\\
s'_t &= s_\bot\langle s''_t\rangle. && (4)
\end{aligned}$$

■

The group distributed justified perspective follows everyone's observations and uses the retrieval function R to identify the values of the variables that are or were not seen by any agent in the group. Intuitively, given any agent i in the group, the value from i's observation in timestamp t, $O_i(s_t)$, which leads to knowledge, must be true (Axiom T) in s_t. While the value of an unseen variable is determined by anyone in the group who saw it last. To be specific, the last timestamp the group sees v, lt_{tv}, is determined by the group observation (formed by union). The value e is then retrieved by identifying the closest value that is consistent with it. Line (3) ensures the "group memory" is consistent with the group observation, while Line (4) ensures that the group justified perspective is a sequence of complete states (by filling missing variables with a none value $\bot$).

Example 2. Let variables be $V = \{x, y\}$, domains be $D_x = D_y = \{1, \ldots, 6\}$, and a state sequence be $\overrightarrow{s} = [s_0, s_1, s_2] = [1\text{-}2, 3\text{-}4, 5\text{-}6]$. Assume a sees x and y in s_0, while b sees y in s_1 and c sees x in s_2. So, $O_a(\overrightarrow{s})$=[1-2, _-_, _-_], $O_b(\overrightarrow{s})$=[_-_, _-4, _-_] and $O_c(\overrightarrow{s})$=[_-_, _-_, 5-_]. This is visualized in Figure 2.

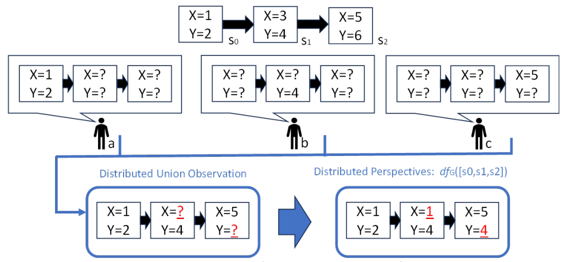

Fig. 2: State sequence $\overrightarrow{s}$ and $df_G(\overrightarrow{s})$ in Example 2

Common belief is the infinite nesting of beliefs. Our definition avoids having the infinite regression by finding the fixed point (set) of the group's perspectives.

Definition 2 (Common Justified Perspectives). *Given a set of perspectives (that is, a set of sequences of states) $\overrightarrow{S}$, the common justified perspective is defined as follows:*

$$cf_G(\overrightarrow{S}) = \begin{cases} \bigcup_{\overrightarrow{s}\in\overrightarrow{S}} ef_G(\overrightarrow{s}) & \textit{if } \bigcup_{\overrightarrow{s}\in\overrightarrow{S}} ef_G(\overrightarrow{s}) = \overrightarrow{S} \\ cf_G(\bigcup_{\overrightarrow{s}\in\overrightarrow{S}} ef_G(\overrightarrow{s})) & \textit{otherwise.} \end{cases} \qquad \blacksquare$$

The function applies a set union on the uniform perspectives of the group for each input perspective. Then, the common perspective function repeatedly calls itself by using the output of one iteration as the input of the next iteration, until the input set and output set are the same, which means a convergence of the common perspectives. Semantically speaking, each iteration adds one level deeper nested perspectives of everyone's uniform belief for evaluation on whether everyone in the group believes.

Definition 3 (Ternary Semantics for Common Belief). *The group ternary semantics are defined using function T, omitting the model M for readability:*

$$(r){:}\quad T[\overrightarrow{s}, CB_G\varphi] = \min(\{T[\overrightarrow{g}, \varphi] \mid \overrightarrow{g} \in cf_G(\{s_\perp\langle\overrightarrow{s}\rangle\})\}) \qquad \blacksquare$$

The common justified perspectives function cf_G contains the fixed point of all agents' perspectives, their perspectives about others' perspectives, and so on to infinite depth. Although the depth is infinite, the definition of cf_G converges (proved) in finite iterations ($2^{|V|\times|\overrightarrow{s}|}$). Practically, in our experiments, we find that it converges after a few iterations (Section 4).

Example 3. Let us use the plan in Figure 1. A state is represented by whether a and b are peeking and the value of the number. The sequences of the states can be represented by a list of states as: $\overrightarrow{s} = [\text{f-f-2}, \text{t-f-2}, \text{f-f-2}, \text{f-f-1}, \text{f-t-1}]$.

Then, the common justified perspective $cf_G(\{\overrightarrow{s}\})$ of a group G in the above example converged within 3 iterations. The output is $\{[\text{f-f-}\perp, \text{t-f-2}, \text{f-f-2}, \text{f-f-2}, \text{f-t-2}], [\text{f-f-}\perp, \text{t-f-}\perp, \text{f-f-}\perp, \text{f-f-}\perp, \text{f-t-1}], [\text{f-f-}\perp, \text{t-f-}\perp, \text{f-f-}\perp, \text{f-f-}\perp, \text{f-t-2}]\}$. According to the ternary semantics of the CB operator, $T[\overrightarrow{s}, CB_G n < 3] = \min(\{T[\overrightarrow{g}, n < 3] \mid \overrightarrow{g} \in cf_G(\{\overrightarrow{s}\})\}) = \min(\{1,1,1\}) = 1$. Therefore, a common belief is formed for the group $G = \{a, b\}$ that $CB_G n < 3$.

ID	n in cf^n Max	Avg	$\lvert cf \rvert$ Max	Avg	External $\#cf$	$\#calls$	$\overline{\mathrm{T}}$ (ms)	Total T (s)	Goals
N0	0	0	0	0	0	141	0.10	0.04	$EB_G\ n < 2$
N1	0	0	0	0	0	26	0.13	0.01	$DB_G\ n < 2$
N2	4	2.20	5	2.16	141	141	0.43	0.09	$CB_G\ n < 2$
N3	4	1.87	5	1.86	446	141	0.91	0.15	$CB_G CB_G\ n < 2$
N4	4	1.76	5	1.75	971	141	1.70	0.26	$CB_G CB_G CB_G\ n < 2$
N5	3	2.13	4	2.04	112	112	0.42	0.07	$\neg EB_G\ n{=}1 \wedge \neg EB_G\ n{=}2 \wedge CB_G\ n < 2$
N6	3	1.65	3	1.65	356	178	0.48	0.12	$B_a CB_G\ n{=}2 \wedge B_b CB_G\ n{=}1$
G0	4	3.03	7	5.36	36	41	2.76	0.13	$CB_G\ sct_a{=}t$
G1	4	2.10	7	2.69	229	41	7.51	0.32	$CB_G CB_G\ sct_a{=}t$
G2	4	1.79	7	2.17	652	41	16.00	0.67	$CB_G CB_G CB_G\ sct_a{=}t$
G3	4	1.66	7	1.99	1451	41	32.78	1.36	$CB_G CB_G CB_G CB_G\ sct_a{=}t$
G4	4	3.38	10	6.74	1474	1860	5.52	11.12	$EB_G\ sct_a{=}t \wedge \neg CB_G\ sct_a{=}t$
G5	5	3.50	14	8.30	8997	13350	10.63	151.92	$EB_G EB_G\ sct_a{=}t \wedge \neg CB_G\ sct_a{=}t$
G6	4	3.02	8	4.21	1828	1138	4.74	5.91	$B_b CB_G\ sct_a{=}f \wedge CB_{\{a,c,d\}} sct_a{=}t$
G7	4	2.66	8	3.18	14800	3792	10.23	40.72	$CB_{\{b,c\}} CB_G\ sct_a{=}f \wedge CB_{\{a,d\}} sct_a{=}t$
G8	4	3.38	10	6.74	1474	1860	6.84	13.61	$DB_G EB_G\ sct_a{=}t \wedge \neg CB_G\ sct_a{=}t$
G9	0	0	0	0	0	1926	1.23	3.31	$DB_G EB_G\ sct_a{=}t \wedge \neg B_a EB_G\ sct_a{=}t$

Table 1: G represents the group of all agents."n in cf^n" are the number of iterations for each cf, and $|cf|$ represent the size of converged cf. Under "External": "$\#cf$"represents the number of cf function calls, while "$\#calls$" and "$\overline{\mathrm{T}}$ (ms)" are the number and average time of external function calls. "T (s)" is total runtime.

4 Experiments and Results

Since there are no planning benchmarks for group belief, we select two domains (Number and Grapevine) from existing work [13, 14] and add several challenging instances that use nested group beliefs.

The results can be found in Table 1. All group beliefs, except for common belief, can be evaluated easily ($\overline{\mathrm{T}}$ in N0, N1, and G6). The number of iterations for cf to converge is around $1.65-3.50$ – much less than the worst-case $2^{|V|\times|\overrightarrow{s}|}$. This is because, in practical epistemic benchmarks, the number of justified perspectives is bounded by the actual state sequences and the difference between each single agent's nested perspectives, resulting in relatively small converged sizes (as shown by $|cf|$). Specifically, N2-N4 and G0-G4 have the exact same search trees (as their goals are semantically equivalent due to CB satisfying Axiom **4**), while when reasoning about nested common belief (using G2 as an example), the growing rate of $\#cf$ (excluded $\#cf$ in G0 and G1) is approximately the average of $|cf|$ for the lower nesting depth (for $\#cf$ in G2: $229+(229-36)\times\overline{|cf^2|} = 651.67 \approx 652$, where $\overline{|cf^2|} = (229\times 2.69-36\times 5.36) \div (229-36) = 2.19$) instead of the number of agents. This growing rate often keeps decreasing (still ≥ 1). The only exception is when all agents see (know) everything, which results in the common perspective sets only containing the actual global perspective for all sequences ($cf(\overrightarrow{s})=\{\overrightarrow{s}\}\rightarrow\overline{|cf^n|}=1$).

Semantically, it is worth noting that everyone beliefs do not result in a common belief (G4). Even if everyone believes that a's secret is true, it does not form a common belief that s's secret is true (G5).

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